Encryption Details

COMP620

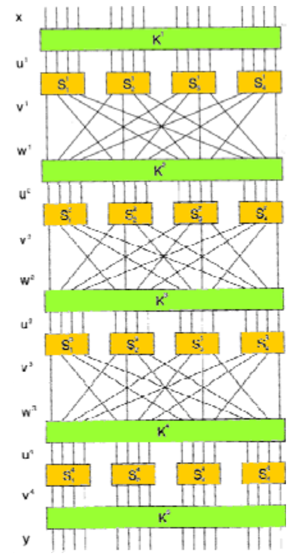
Goals for Today

- Understand how some of the most common encryption algorithms operate
- Learn about some new potential encryption systems

Substitution Permutation Ciphers

- A Substitution Permutation encryption algorithm typically involves three phases, which are often repeated
- **Substitution** – the substitution of a bit pattern with another
- **Permutation** – the rearrangement of the bits
- **Exclusive OR** with a key

Substitution Permutation stages



- The K box XORs the input with the key for that round
- The S box performs a substitution

diagram from “Cryptography Theory and Practice”, 3rd ed. by Douglas Stinson

S Box

- An S box performs a substitution.
- The substitution can be efficiently implemented by a look up table
- Example of a 3 bit to 3 bit substitution

input	000	001	010	011	100	101	110	111
output	101	010	110	000	111	001	011	100

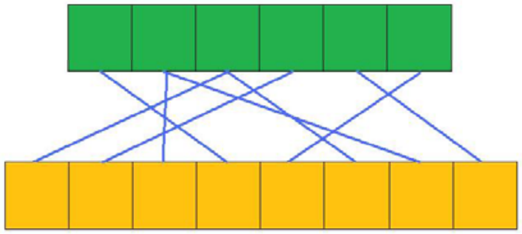
S Box Expansion or Contraction

- The number of bits on the input of an S box does not have to match the number of bits on the output
- Example of a 3 bit input with 2 bit output

input	000	001	010	011	100	101	110	111
output	10	01	11	01	11	00	10	00

Permutation Expansion

- The number of bits on the output of a permutation does not need to match the input
- Some input bits can go to multiple output bits



Stage Keys

- The key used at each stage is a function of the original key
- Before each stage the key is modified to produce a unique key for that stage
- Some stages might use only some of the key bits

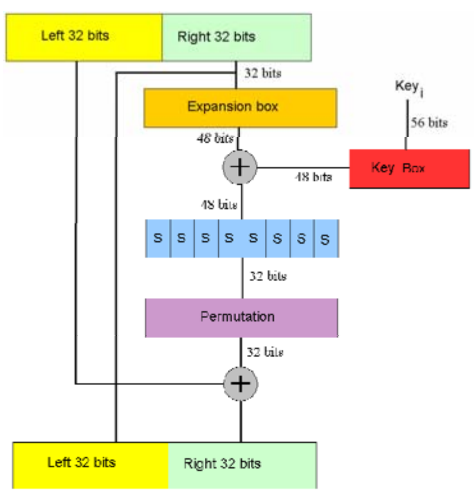
Data Encryption Standard

- Originally developed by IBM
- Adopted as a standard in 1977
- Was the most widely used cryptosystem in the world
- DES uses a 56 bit key
- 64 bit blocks of data are encrypted

DES Algorithm

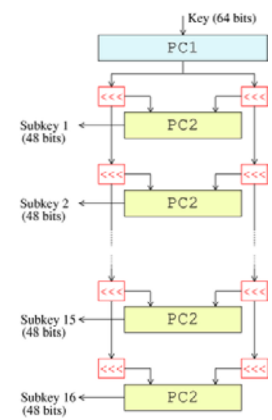
- DES is a substitution permutation cipher
- There are 16 stages
- The data is split into the left and right half. Each 32 bit half is handled differently
- The 56 bit key is divided into two 28 bit halves which are used to create unique 48 bit keys for each stage

DES Stage



DES Key Schedule

- The 56 bit key is split into two 28 bit halves
- Each half is rotated 1 or 2 bits to the left
- 48 of the 56 bits are selected for the stage key
- Each bit is used in 14 of the 16 stages



DES Effectiveness

- Analysis has found few weaknesses in DES
- Differential cryptanalysis can find the key with 2^{47} chosen plaintext/ciphertext pairs
- The DES key is too short
- Brute force attacks can defeat DES

Triple DES

- To improve security while still using DES, triple DES is usually used.

  $\text{ciphertext} = E_{Key3}(D_{Key2}(E_{Key1}(\text{plaintext})))$
- The three keys can be:
  - All different – best security using 168 bit keys
  - Key1 = Key3, Key2 different – Good with 112 bit keys
  - All identical – same as single DES

Advanced Encryption Standard

- AES is also known as the *Rijndael* algorithm
- Selected in 2000 as the new standard after an open international competition
- Created by Belgian researchers Rijmen and Daemen
- Available world-wide royalty free
- AES encrypts blocks of 128 bits
- Keys can be either 128 bits, 194 bits or 256 bits
- AES operates on a 4×4 array of bytes, termed the *state*

AES Algorithm

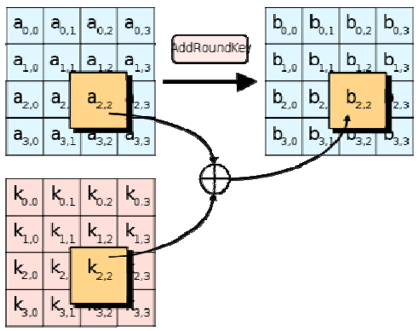
Initial Stage
- AddRoundKey

Each Stage
- SubBytes—a substitution step
- ShiftRows—rows are shifted cyclically
- MixColumns—each column of the state is multiplied with a fixed polynomial
- AddRoundKey—each byte is XOR with the stage key

Final Stage(no MixColumns)
- SubBytes
- ShiftRows
- AddRoundKey

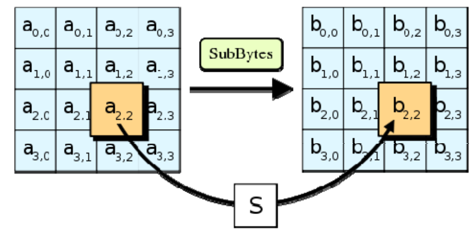
AddRoundKey Step

- Each byte of the data is XOR with the key for that stage



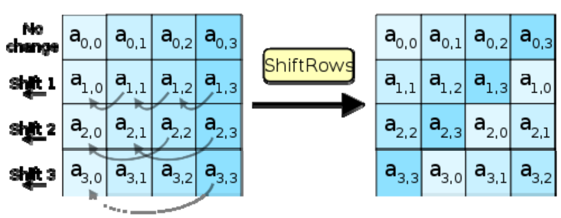
SubBytes Step

- Each byte in the array is updated using an 8 bit substitution box, the Rijndael S-box.



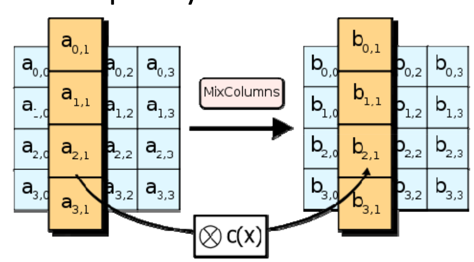
ShiftRows Step

- This cyclically shifts the bytes in each row by a certain offset.



MixColumns Step

- Each column is multiplied by a fixed polynomial. All four input bytes determine the four output bytes.



AES Implementation

- It is possible to speed up execution of this cipher by combining SubBytes and ShiftRows with MixColumns, and transforming them into a sequence of table lookups
- This requires 4KB of lookup tables
- A stage can now be done with 16 table lookups and 12 XOR, followed by four XOR in the AddRoundKey step
- Intel and AMD are adding instructions to the Pentium to perform an AES stage

RSA Algorithm

- RSA is an asymmetric encryption algorithm developed in 1977 by Rivest *(author of COMP785 text)*, Shamir and Adleman
- Secretly developed by Clifford Cocks in 1973
- The most commonly used public key algorithm
- Now in the public domain

RSA Encryption

- The sender encrypts message m as

$$c = m^e \bmod n$$

- ***c*** is sent to the receiver
- The receiver computes

$$m = c^d \bmod n$$

RSA Algorithm

- Select secret primes $\boldsymbol{p}$ and $\boldsymbol{q}$ and form $\boldsymbol{n} = \boldsymbol{p}*\boldsymbol{q}$
- The totient $\Phi(\boldsymbol{n}) = (\boldsymbol{p}-1)*(\boldsymbol{q}-1)$
- Choose $\boldsymbol{e}$ with $gcd(\boldsymbol{e}, \Phi(\boldsymbol{n})) = 1$
- Compute $\boldsymbol{d}$ such that $\boldsymbol{d}*\boldsymbol{e} = 1 \bmod \Phi(\boldsymbol{n})$
- The public key is $\boldsymbol{n}$ and $\boldsymbol{e}$
- The secret key is $\boldsymbol{d}$

RSA Example

- Let ***p*** = 101 and ***q*** = 113
- ***n*** = 11413 = ***p*** * ***q***
- Φ(***n***) = (***p***-1)*(***q***-1) = 11200
- Choose ***e*** = 3533 (gcd of ***e*** and Φ(**n**) is 1)
- ***d*** = e^{-1} mod 11200 = 6597
- Assume ***m*** = 9726
- Encryption: ***c*** = 9726^{3533} mod 11413 = 5761
- Decryption: ***m*** = 5761^{6597} mod 11413 = 9726

Sizes

- Typically large values of ***p*** and ***q*** are selected to make it difficult to factor.
- Therefore ***n*** is very large, $\log_2 n$ bits in length
- The pieces of plain text to encrypt, ***m***, must be $0 < m < n$
- The cipher text can be up to $\log_2 n$ bits long

Another RSA Example

- Let ***p*** = 885320963 and ***q*** = 238855417
- ***n*** = 211463707796206571
- Choose ***e*** = 9007
- ***d*** = e^{-1} mod 11200 = 116402471153538991
- Assume **m** = 30120
- ***c*** = 113535859035722866
- Note that ***c*** is much larger than ***m***
- It requires more space to store ***c*** than ***m***

RSA Key Concern

- For ***n*** = ***p*q***, if we know the first or last ($\log_2$**n**)/4 digits of ***m***, we can efficiently factor ***n***
- If ***p*** has 100 digits, then if we know the first or last 50 bits of ***p*** or ***q***, we can factor ***n***
- Imagine we select **p** and **q** by picking a random 50 bit number and multiplying it by 2^{50} to get 100 bits. We then add 1 and test for primality until we get a prime
- We can guess the lowest 50 bits of the number

Short Theory Review

- Remember that algorithms can be classified according to the time (or memory) required for the solution of an ***n*** sized problem

$O(2^n)$	exponential	
$O(n^x)$		polynomial
$O(n^2)$		
$O(n \log n)$		
$O(n)$	linear	
$O(\log n)$		
$O(1)$	constant	

Oracles

- In algorithm theory we assume we have an oracle that can guess solutions and will always guess correctly.
- If you want to find a path through all nodes of a graph, you could ask the oracle what is the next node to traverse.
- It would take ***n*** steps to find the shortest path through an ***n*** node graph.

Verifying the Solution

- Once you have a solution from an oracle, you can verify that it is correct.
- If you can verify that the solution is correct in polynomial time, the problem is **NP-hard**

Non-deterministic

Polynomial time

Known NP Problems

- **Subset Sum problem** – given a set of integers {−7, −3, −2, 5, 8} is there a subset that will sum to zero?
- **Knapsack problem** – Given a set of items, each with a weight and a value, what is the optimal set to put in a fixed sized box?
- **Boolean Satisfiability problem** – can the variables of a logical expression be given values that will make the expression true?
- **All polynomial time problems**

Problem Reduction

- If you can convert (or reduce) problem A to problem B, then you know if you can solve problem B, you can always solve problem A
- The subset sum problem can be reduced to the knapsack problem

NP-Complete

- If you can reduce a known NP-hard problem to problem X, then if you can solve problem X in polynomial time, you can solve the NP-hard problem in polynomial time.
- NP-Complete is a set of HP-hard problems that are all reducable. If you can solve one in polynomial time, you can solve all of them.

Some NP-Complete Problems

- Graph coloring
- Traveling salesman
- Bin packing
- Knapsack
- Hamiltonian graphs
- Equivalence of Context Free Languages
- Sudoku
- Boolean satisfiability

Knapsack Problem

- The knapsack algorithm is an NP-Complete problem
- Consider a set of integers $x_0 \ldots x_n$ and a number E which is greater than any x_i
- Can you create a set of integers $a_0 \ldots a_n$ such that

$$E = \sum_{i}^{n} a_i * x_i$$

Easy Knapsack Problem

- The super increasing knapsack algorithm has a linear solution
- Consider a set of integers $x_0 \ldots x_n$ such that each x_i is greater than the sum of all x_i before it.
- It is easy to solve this problem

Knapsack Encryption Algorithm

- R. Merkle and M. Hellman devised an asymmetrical encryption algorithm based on the knapsack problem.
- A super increasing set of x_i is transformed into a regular knapsack set of x_i by a method known only to the secret key holder
- The public key is the normal knapsack set of x_i

Knapsack Encryption

- To encrypt message ***b***, *let* $b=b_1 b_2 \ldots b_n$ *(in binary format)*
- Compute $y= b_1x_1+ b_2x_2 + \ldots + b_nx_n$ *and send* ***y***
- To decrypt, solve for b_i using the super increasing knapsack values
- Flaws have been found in the algorithm

Dr. Williams' Research

- I have been working on developing a new asymmetric encryption algorithm based on a non-computable number
- No algorithm has yet been devised, but there is no reason to believe that a solution does not exist

Harder than Hard

- The RSA algorithm is secure because it is difficult to factor large numbers. The simple algorithm is $O(\sqrt{n})$
- Computers are getting faster and mathematicians are getting smarter
- There are problems that are harder than NP-Complete problems

No Possible Solution

- The halting problem is the best known non-computable problem. You cannot write an algorithm that reads a program and its data and can always tell if the program comes to the end.
- There are many instances of non-computable problems that can be easily solved, but you cannot solve all of them
- There are several known non-computable problems

Post correspondence problem

- Given a set of domino-like tiles with symbols on the top and bottom, can a series of tiles be placed side by side so that the string of symbols on the top matches the string of symbols on the bottom
- Assume you have an infinite number of each type of domino

100	0	1
1	100	00

Solution to Example

Simple solution to this instance of the problem requiring seven dominos

100	1	100	100	1	0	0
1	00	1	1	00	100	100

More Difficult Problem

- If you change the bottom number on the last tile so that it only contains one zero, the problem becomes much more difficult.
- This simple change has a solution, but it requires 75 dominos.

100	0	1
1	100	0

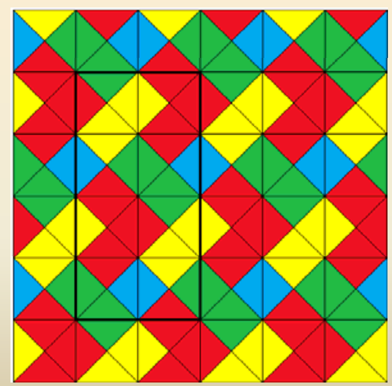
Difficulty

- How do you know if there is no solution?
- With the previous problem, had you tried all possible combinations of 74 or less dominos, you would not have found an answer, but one does exist
- There still exists sets of 3 dominos with 3 or less digit binary numbers where it is unknown if a solution exists

Wang tiles

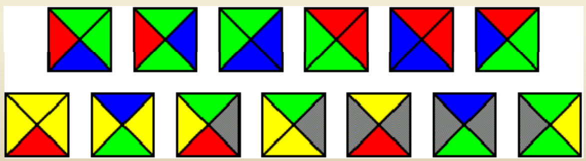
- When using a set of square tiles with colored edges, can the tiles be arranged without rotation or reflection so that they tile a plane with adjacent tiles having edges of the same color?
- Assume you have an infinite supply of each tile

Tiling Solution

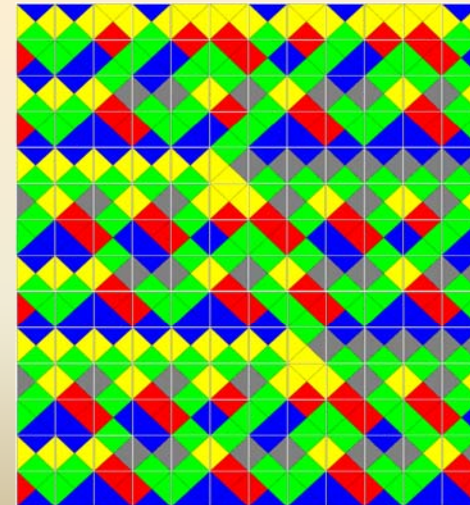


Periodic and Aperiodic

- The previous solution has a 2 by 4 block that can be repeated forever
- Some sets of tiles can tile an infinite plane without repeating

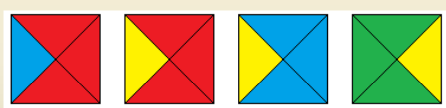


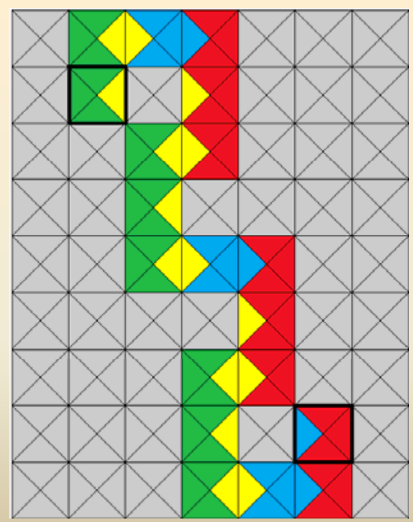
Aperiodic Tiling



Domino Snakes

- Using tiles similar to Wang Tiles, can a given set of tiles form a path from two given points so that all adjacent tile edges have matching colors?





Snake Solution

Use in Encryption

- Like the knapsack encryption algorithm, we are looking for a way to transform the tiles from a nearly impossible problem to a simple problem.
- We have been developing heuristics to identify sets of tiles that do or do not tile the plane or form a snake