Finite State Automata

COMP360

“It has been said that man is a rational animal. All my life I have been searching for evidence which could support this.”

Bertrand Russell

Reading

Read sections 2.1 – 2.4 in the textbook

Assembler Assignment

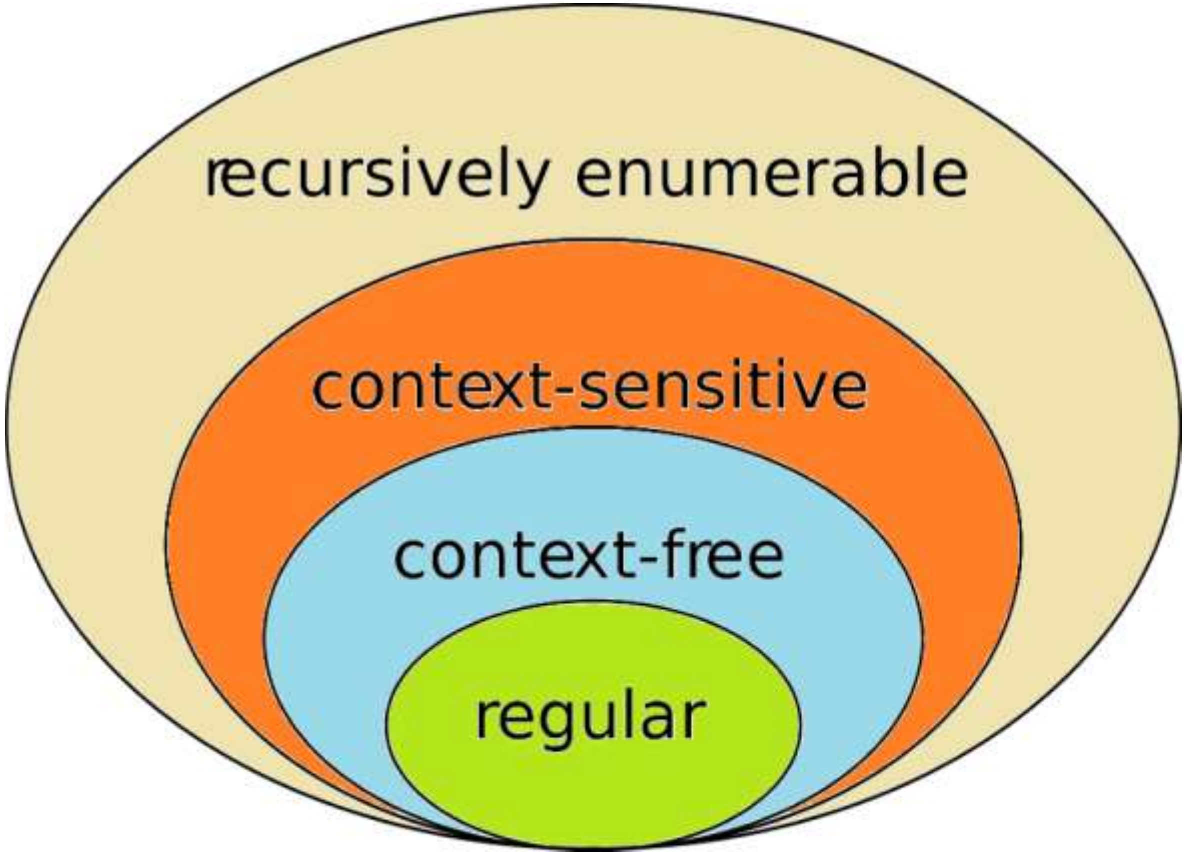
- The assembler program is due before we meet again
- Upload the source code of your assembler to Blackboard before 1:00 on Monday, January 27, 2020

Language Recognition

- We say a theoretical machine can recognize a language if it can correctly identify a string of characters as being a syntactically correct program
- If a theoretical machine can recognize a language, it can identify input that is not in the proper form, i.e. missing a semicolon or improper brackets

Chomsky Hierarchy

- There are four classes of languages
- Each class is a subset of another class
- Regular languages are the simplest while recursively enumerable (RE) languages are the most complex



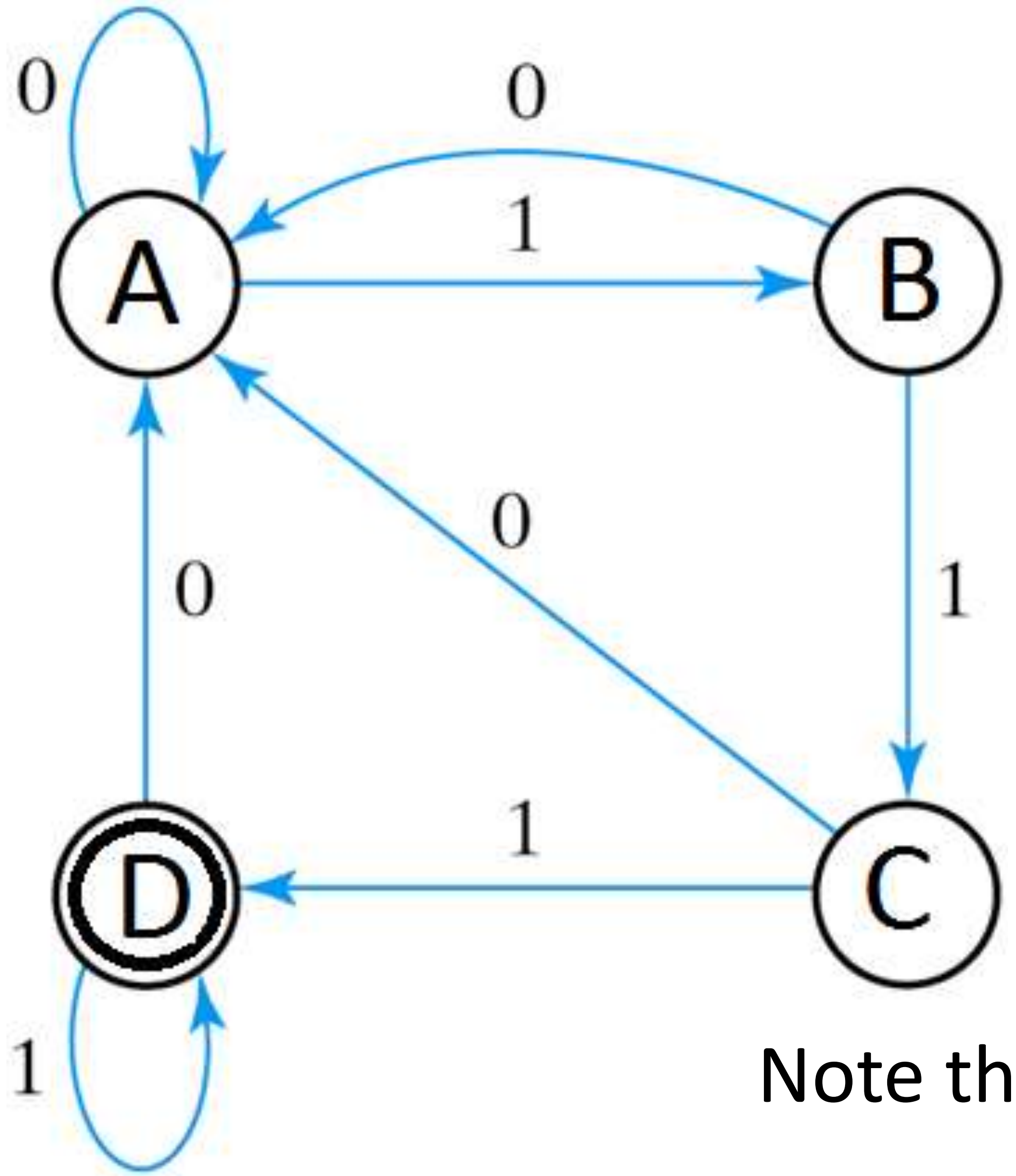
Languages and Machines

Language	Machine	Examples
Regular Expressions	Deterministic Finite State Automata (DFA)	a b* a
Context Free	Push Down Automata (PDA)	$a^n b^n$
Context Sensitive	Bounded Turing Machine	$a^n b^n c^n$
Recursively Enumerable	Turing Machine	Anything that can be recognized

Finite State Automata

- A FSA has circles to represent states and arrows to represent transitions between states
- Arrows are labeled with a character. That transition is taken if the FSA is in that state and the next input character matches the arrow label
- There is one or more final states (double circle) that indicate the input matches the language
- If there is no matching transition for an input, the FSA rejects the input as a member of the language

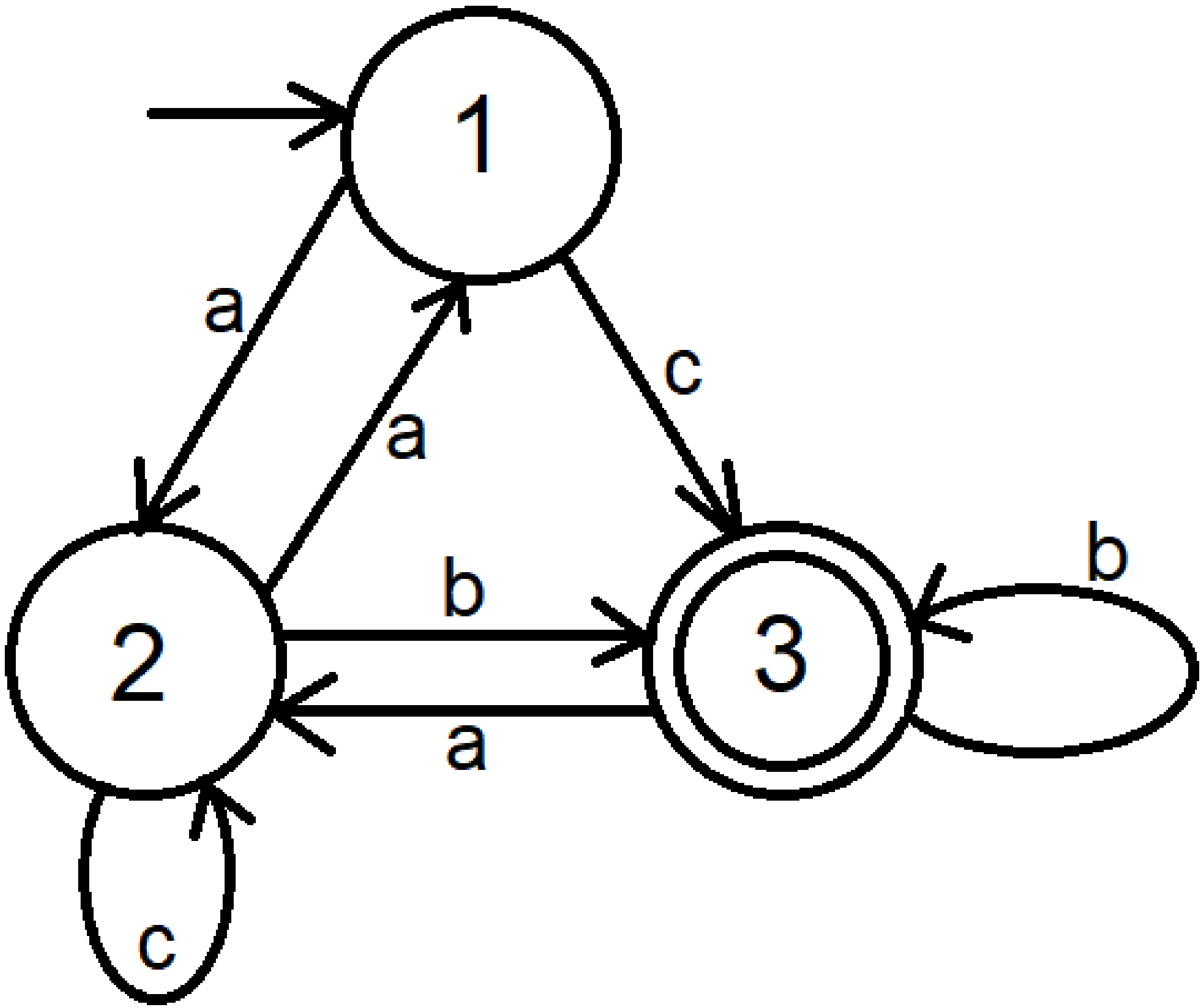
FSA for ending in 3 Consecutive 1's



- State A : no 1s detected
- State B : one 1 detected
- State C : two 1s detected
- State D : three 1s detected

Note that each state has 2 output arrows

What string is NOT accepted by this FSA?



A. accb
B. aac
C. aab
D. c
E. cbab

Nondeterministic FSA

- In Computer Science theory, a Nondeterministic FSA (NFSA) can have multiple out arrows with the same label
- An NFSA magically guesses which transition to take
- Since an NFSA is no more powerful than a deterministic FSA *(and we don't have magical programs)* we will only use deterministic FSA
- Our FSA cannot have duplicate out arrows with the same label

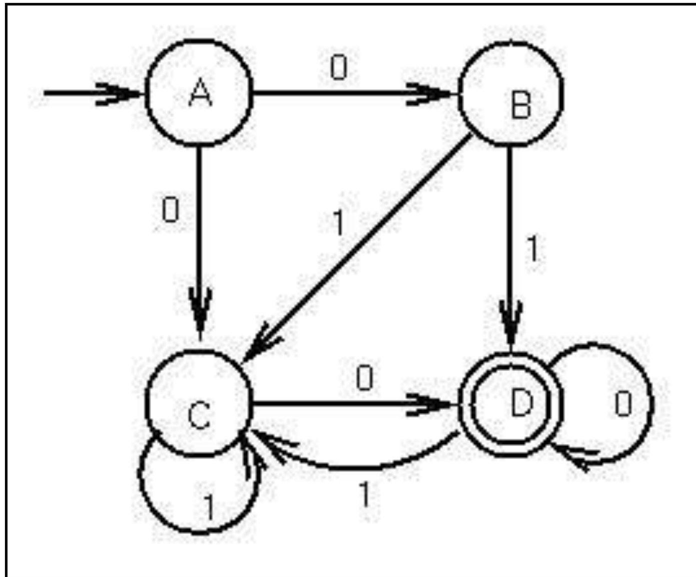
Deterministic FSAs

- Deterministic FSA: For each state and for each input symbol, there is exactly one transition
- Non-deterministic FSA (NDFSA): Remove this restriction
- At each node there is 0, 1, or more than one transition for each alphabet symbol
- A string is accepted if there is *some* path from the start state to some final state
- Example nondeterministic FSA (NDFSA):

01 is accepted via path: ABD

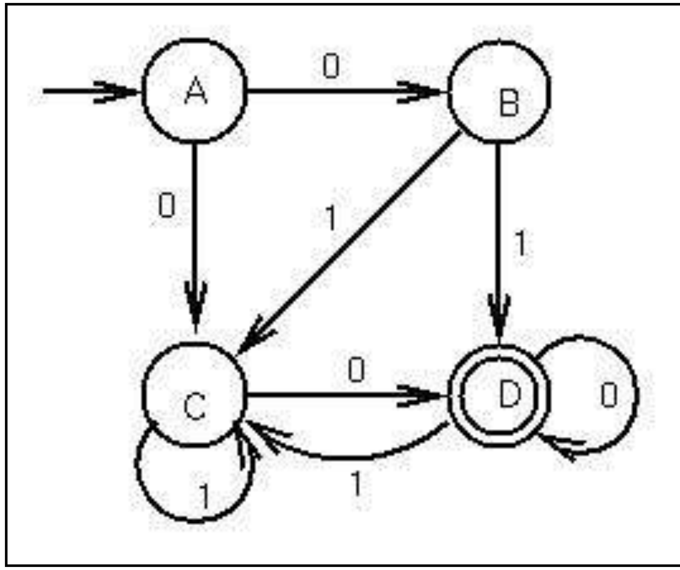
even though 01 also can take the paths:

ACC or ABC where C is not a final state



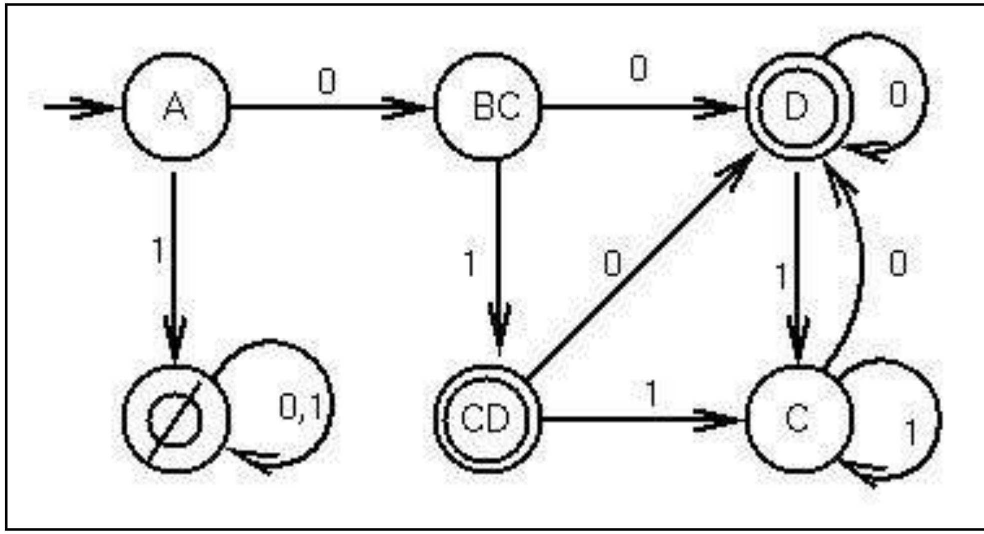
Equivalence of FSA and NDFSA

- Important early result: NDFSA = DFSA



- Let subsets of states be states in DFSA.
- Keep track of which subset you can be in.

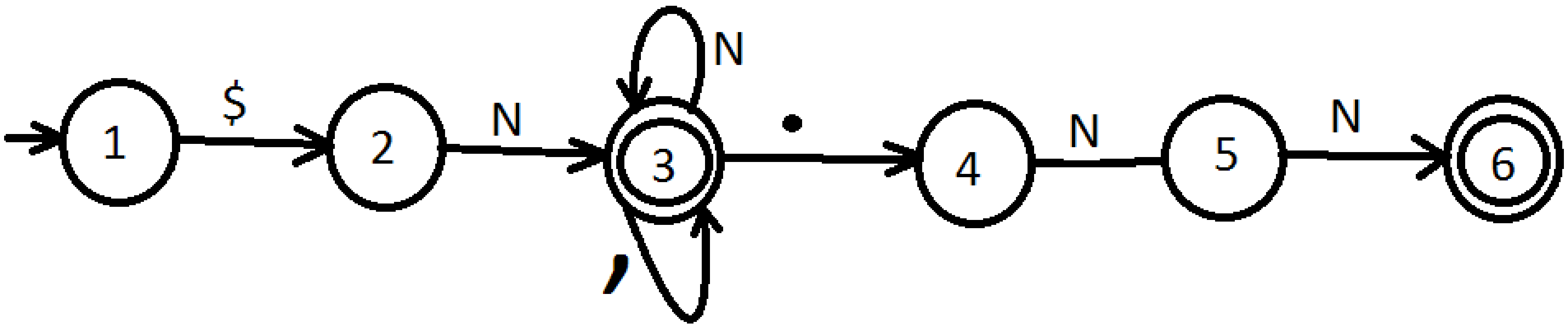
- Any string from {A} to either {D} or {CD} represents a path from A to D in the original NDFSA.



Draw a FSA

- Draw a FSA the accepts money, such as $123 or $665.95

Possible Solution



Regular Expressions

- Regular expressions are often used to specify a regular language
- A regular expression defines a series of symbols in the order they can appear in the language
- Consider a very simple language with only the letters "**`a`**" and "**`b`**"
- **`aabb`** is a regular expression for a string with two letters **`a`** followed by two **`b`**'s

Regular Expression Alternatives

- A regular expression can specify alternatives using the vertical bar character, |
- **`aabb | ba`** defines a regular language that accepts two **`a`**’s then two **`b`**’s or a string with just a **`b`** and an **`a`**
- You can use parenthesis to group strings
- **`(aabb | ba)a`** defines a regular language that accepts **`aabba`** or **`baa`**

Regular Expression Repetitions

- A symbol or (group) might repeat multiple times in a valid string of the language
- A "*" indicates that a symbol or (group) repeats zero, one or many times
- A "+" indicates that a symbol or (group) repeats once or many times, but at least once
- **`a`**$^{0..1}$ mean the symbol may appear zero or 1 time
- **`(aabb | ba)`**$^{+}$ **`a`** defines the strings

  `aabbaabba, aabbbaa, baa` and more

Which string is **NOT** `(aa | bb)*` a^+

A. a
B. bbaabba
C. aaa
D. aabb
E. bbbbbbaaaaa

Write a Regular Expression

- Write a regular expression to define a string that begins and ends with an **`a`** and can have an even number of **`b`**'s between them

Possible Solution

- Write a regular expression to define a string that begins and ends with an **a** and can have an even number of **b**'s between them

a (bb)* a

Regular Expressions for Searching

- Regular expressions are often used to define a search target
- The Unix grep utility (**g**lobally search a **r**egular **e**xpression and **p**rint) uses regular expressions
- The Java Pattern class uses regular expressions to define the pattern

Equivalent

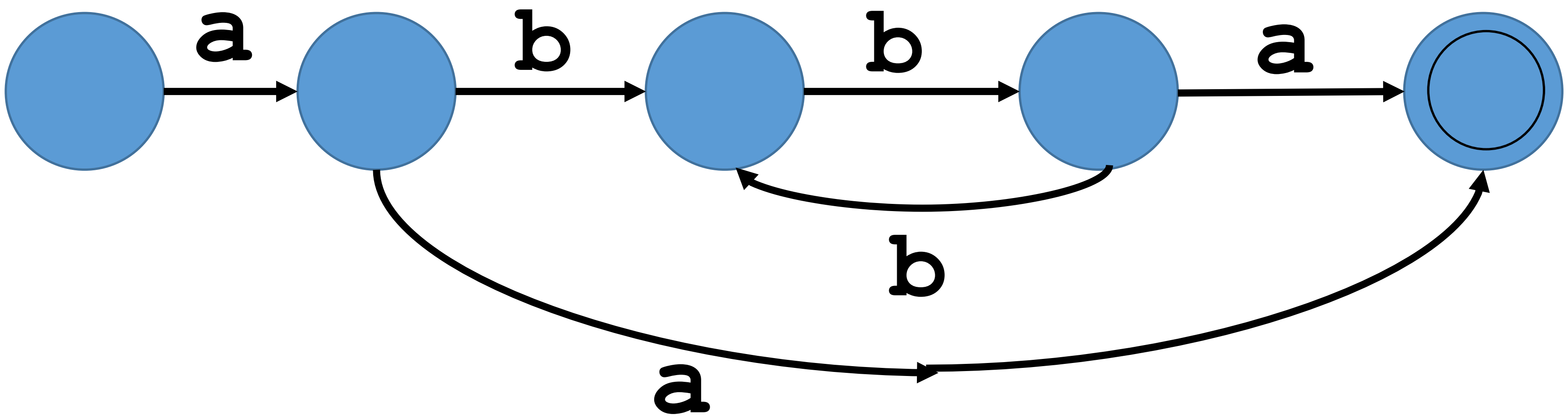
- A FSA and a regular expression can define a regular language
- Regular expressions can be recognized by a regular language

Converting a Regular Expression to a FSA

- Consider the regular expression

  `a (bb)* a`

- It can be recognized by the FSA



Try it

- Write a regular expression that defines a double number, such as 12.3 or -12.3
- Use **`n`** to represent a numerical digit, 0..9

Possible Solution

- Write a regular expression that defines a double number, such as 12.3 or -12.3
- Use **n** to represent a numerical digit, 0..9

$$-^{0..1}\ n^{+}\ .\ n^{*}$$

Degenerate Case

- 123. and .123 are both valid numbers
- A single period is not a valid number
- There must be a digit before or after the decimal point
- A possible solution might be

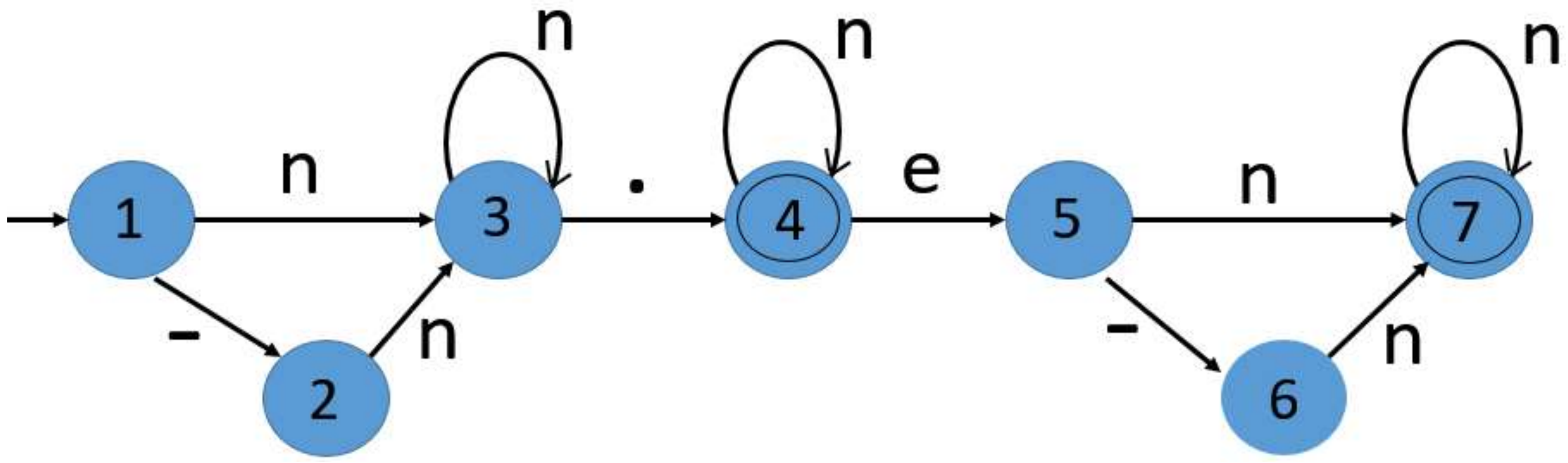
$$-^{0..1}\ ((n^{+}\ .\ n^{*})\ |\ (n^{*}\ .\ n^{+}))$$

With Scientific Notation

- Write a regular expression that defines a double number, such as 12.3 or -1.23e1
- Use **n** to represent a numerical digit, 0..9

$$-^{0..1}\ n^{+}\ .\ n^{*}\ (\ e\ -^{0..1}\ n^{+}\ )^{0..1}$$

FSA to Recognize a Double Number

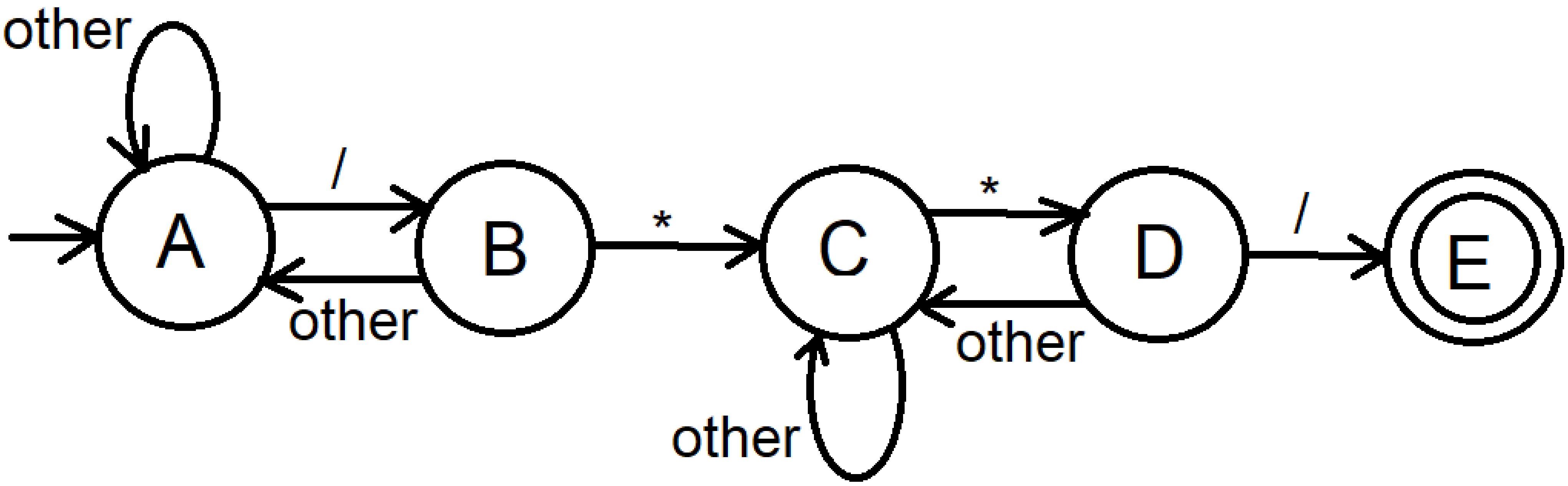


Draw a FSA

- Draw a Finite State Automata that recognizes comments in the **/* form */**
- Use “*other*” to represent any other character

Possible Solution

- Draw a Finite State Automata that recognizes comments in the **/* form */**
- Use "*other*" to represent any other character



FSA Limits

- The only “memory” that a FSA has is the current state
- As a FSA inputs symbols, it moves to different states
- A FSA cannot count or compare an arbitrary number of symbols

Push Down Automata

- A PDA has a FSA and a stack memory
- The top of the stack, input symbol and FSA state determine what a PDA will do
- A PDA can:
  - Push a new symbol on the stack
  - Pop the top symbol from the stack
  - Change the FSA state

Push Down Automata Languages

- A Push Down Automata (PDA) can recognize context free grammars
- Almost all modern programming languages are context free grammars
- A PDA can "count" things
- Nobody has developed a programming language more complicated than a context free grammar

Defining a Language

- For a program to be able to recognize a member of a language, you have to be able to accurately and unambiguously define the language
- A regular language can be defined by a regular expression
- A context free language can be defined by a BNF

Compiler’s Purpose

- A compiler converts program source code into a form that can be executed by the hardware
- A compiler works with the language libraries and the system linker

Stages of a Compiler

- Source preprocessing
- Lexical Analysis (scanning)
- Syntactic Analysis (parsing)
- Semantic Analysis
- Optimization
- Code Generation
- Link to libraries

Source Preprocessing

- In C and C++, preprocessor statements begin with a #
- The preprocessor edits the source code based on the preprocessor statements
- **`#include`** is the same as copying the included file at that point with the editor
- The output of the preprocessor is expanded source code with no # statements
- Old C compilers had a separate preprocessor program

Lexical Analysis

- Lexical Analysis or scanning reads the source code (or expanded source code)
- It removes all comments and white space
- The output of the scanner is a stream of tokens
- Tokens can be words, symbols or character strings
- A scanner can be a finite state automata

I have a laptop with an I5, not a FSA

- You can write a simple program to simulate a FSA
- The simple program can create a list of tokens from source code input
- Simulating a FSA is a very easy way to implement a scanner

Syntactic Analysis

- Syntactic Analysis or parsing reads the stream of tokens created by the scanner
- It checks that the language syntax is correct
- The output of the Syntactic Analyzer is a parse tree
- The parser can be implemented by a context free grammar stack machine

Semantic Analysis

- The Semantic Analysis inputs the parse tree from the parser
- Semantic Analysis checks that the operations are valid for the given operands *(i.e. cannot divide a String)*
- This stage determines what the program is to do
- The output of the Semantic Analysis is intermediate code. This is similar to assembler language, but may include higher level operations

Output of Compiler Stages

Step	Output
• Lexical Analysis (scanning)	List of tokens
• Syntactic Analysis (parsing)	Parse tree
• Semantic Analysis	Intermediate code
• Optimization	Better Intermediate code
• Code Generation	Machine language

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